Business Problems

$p(x)$ - demand function

$p$ is the number sold at a price $x$

$R(x) = xp(x) \rightarrow$ revenue function

$C(x) \rightarrow$ cost function

$P(x) = R(x) - C(x) \rightarrow$ Profit
Marginal

\[
\frac{dC}{dx} = \text{marginal cost}
\]

\[
\frac{dR}{dx} = \text{marginal revenue}
\]

\[
\frac{dP}{dx} = \text{marginal profit}
\]
A store sells 250 pairs of Brand X running shoes each month when priced at $100 per pair. It has been determined that for every $5 decrease in price, an additional 20 pairs will be sold.

a) Determine the demand or price function.

b) Determine the revenue function.

c) Find the revenue when sales are 250 pairs and 290 pairs of shoes per month.

d) Determine the marginal revenue when sales are 290 pairs of shoes per month.

\[ x = 250 + 20n \]
\[ p = 100 - 5n \]
\[ \frac{x - 250}{20} = n \]
\[ p = 100 - 5\left(\frac{x - 250}{20}\right) \]
\[ p = 100 - \frac{x}{4} + \frac{250}{4} \]
\[ p = \frac{650}{4} - \frac{x}{4} \]

\[ R(x) = x \cdot p(x) \]
\[ R(250) = \frac{650}{4} \cdot \frac{-x^2}{4} + \frac{x}{4} \]
\[ R(290) = \]
1. A store sells 250 pairs of Brand X running shoes each month when priced at $100 per pair. It has been determined that for every $5 decrease in price, an additional 20 pairs will be sold.
   a) Determine the demand or price function.
   b) Determine the revenue function.
   c) Find the revenue when sales are 250 pairs and 290 pairs of shoes per month.
   d) Determine the marginal revenue when sales are 290 pairs of shoes per month.

\[
\frac{dR}{dx} = \frac{650}{4} - \frac{x}{2}
\]

\[
R(280) = \frac{650 \cdot (280)}{4} - \frac{280}{4}
\]

\[
R(250) = 25000
\]

\[
R(290) = 26100
\]

\[
\left. \frac{dR}{dx} \right|_{x=290} = \frac{650}{2} - \frac{290}{2} = 17.5
\]
The cost function of producing yo-yos is 

\[ C(x) = 1400 + 0.01x + 0.001x^2. \]

**a)** Find the marginal cost of producing 3000 yo-yos.

**b)** The revenue from selling \( x \) number of yo-yos is \( R(x) = 6x \). Determine the marginal profit from selling 3000 yo-yos.

\[
C'(x) = 0.01 + 0.002x
\]

\[
C'(3000) = 0.01 + 0.002(3000)
\]

\[ = 6.01 \]

\[
P'(x) = R'(x) - C'(x)
\]

\[
= 6 - (1400 - 0.01x - 0.001x^2)
\]

\[ = -0.01 \]
The volume of water in a tank that is draining from the bottom is given by the

function \( V(t) = 150 \left( 1 - \frac{t}{5} \right)^2 \), \( 0 \leq t \leq 5 \), where

\( V \) is in litres and \( t \) is in minutes.

a) At what rate is the water draining from the tank after 2 min? after 3 min?

b) When is the water completely drained from the tank? Explain how you determined this.

\( V = 0 \)

\[ V' = 150(2)(1 - \frac{t}{5}) \left( -\frac{1}{5} \right) = -60(1 - \frac{t}{5}) \]

\[ V'(2) = -36 \text{ litres/min} \]

\[ V'(3) = -24 \]
The mass, in grams, of a wire is given by the function \( f(x) = x\left(1 + \sqrt{x}\right) \) where \( x \) is the length measured in metres from one end.

a) Determine the mass of the wire when \( x = 5 \).

b) Determine the linear density of the wire when \( x = 2 \) m and when \( x = 5 \) m.

\[
\begin{align*}
f(5) &= 5\left(1 + \sqrt{5}\right) = 5 + 5\sqrt{5} = 16.18 \\
f(2) &= 1 + \frac{3}{2}\sqrt{2} = 3.12 \\
f(5) &= 1 + \frac{3}{2}\sqrt{5} = 4.35
\end{align*}
\]
5. A toy rocket is shot into the air. Its height, in metres, after \( t \) seconds is given by
\[ h(t) = -4.9t^2 + 28t + 1.2. \]

a) Determine the height of the rocket after \( 3 \) s.
\[ h(3) = -4.9(3)^2 + 28(3) + 1.2 = 41.1 \]

b) Determine the rate of change of the height of the rocket after \( 2 \) s and \( 4 \) s.
\[ h'(t) = -9.8t + 28 \]
\[ h'(2) = 8.4 \]
\[ h'(4) = -11.2 \]

c) How long does it take the rocket to hit the ground?

\[ h(t) = 0 = -4.9t^2 + 28t + 1.2 \]
\[ t = \frac{-28 \pm \sqrt{28^2 - 4(-4.9)(1.2)}}{2(-4.9)} \]
\[ t = 5.76 \]
\[ h'(5.76) = -9.8 \times 5.76 + 28 \]
\[ = -28.448 \]

\[ h''(x) = -9.8 \]
Functions Pertaining to Business

- The demand function, or price function, is $p(x)$, where $x$ is the number of units of a product or service that can be sold at a particular price, $p$.

- The revenue function is $R(x) = xp(x)$, where $x$ is the number of units of a product or service sold at a price per unit of $p(x)$.

- The cost function, $C(x)$, is the total cost of producing $x$ units of a product or service.

- The profit function, $P(x)$, is the profit from the sale of $x$ units of a product or service. The profit function is the difference between the revenue function and the cost function: $P(x) = R(x) - C(x)$. 
Derivatives of Business Functions

Economists use the word *marginal* to indicate the derivative of a business function.

- $C'(x)$ or $\frac{dC}{dx}$ is the **marginal cost function** and refers to the instantaneous rate of change of total cost with respect to the number of items produced.

- $R'(x)$ or $\frac{dR}{dx}$ is the **marginal revenue function** and refers to the instantaneous rate of change of total revenue with respect to the number of items sold.

- $P'(x)$ or $\frac{dP}{dx}$ is the **marginal profit function** and refers to the instantaneous rate of change of total profit with respect to the number of items sold.
A store sells 250 pairs of Brand X running shoes each month when priced at $100 per pair. It has been determined that for every $5 decrease in price, an additional 20 pairs will be sold.

a) Determine the demand or price function.

b) Determine the revenue function.

c) Find the revenue when sales are 250 pairs and 290 pairs of shoes per month.

d) Determine the marginal revenue when sales are 290 pairs of shoes per month.
A store sells 250 pairs of Brand X running shoes each month when priced at $100 per pair. It has been determined that for every $5 decrease in price, an additional 20 pairs will be sold.

a) Determine the demand or price function.

b) Determine the revenue function.

c) Find the revenue when sales are 250 pairs and 290 pairs of shoes per month.

d) Determine the marginal revenue when sales are 290 pairs of shoes per month.

\[ R = \frac{650x}{4} - \frac{x^2}{4} \]

\[ R(250) = \frac{650(250)}{4} - \frac{250^2}{4} \]

\[ R(250) = 25000 \]

\[ R(290) = 26100 \]

\[ R'(x) = \frac{650}{4} - \frac{2x}{4} \]

\[ R'(290) = \frac{650}{4} - \frac{2(290)}{4} \]

\[ = 17.5 \]
\[
\frac{R(291) - R(290)}{650(291)} - \frac{291^2}{4} - \frac{650(290)}{4} + \frac{290^2}{4}
\]

= 17.25

\[
R'(320) = -2.5
\]

\[
R(330) = \frac{650(330)}{4} - \frac{330^2}{4} = 26400
\]
The cost function of producing yo-yos is \( C(x) = 1400 + 0.01x + 0.001x^2 \).

a) Find the marginal cost of producing 3000 yo-yos.

\[ P'(3000) = 5.99 - 0.02(3000) = -0.01 \]

b) The revenue from selling \( x \) number of yo-yos is \( R(x) = 6x \). Determine the marginal profit from selling 3000 yo-yos.

\[ C'(x) = 0.01 + 0.002x \]

\[ C'(3000) = 0.01 + 0.002(3000) = 6.01 \]

\[ P = R - C = 6x - 1400 - 0.01x - 0.001x^2 \]

\[ P = 5.99 - 0.002x \]
The volume of water in a tank that is draining from the bottom is given by the function 

\[ V(t) = 150 \left(1 - \frac{t}{5}\right)^2, \quad 0 \leq t \leq 5 \]

where 

- \( V \) is in litres and \( t \) is in minutes.

a) At what rate is the water draining from the tank after 2 min? after 3 min?

b) When is the water completely drained from the tank? Explain how you determined this.

\[ V'(t) = 150 \left(2 \left(1 - \frac{t}{5}\right) \left(-\frac{1}{5}\right)\right) = -60 \left(1 - \frac{t}{5}\right) \]

\[ V'(2) = -36 \text{ litres/minute} \]

\[ V'(3) = -24 \]
$$V(t) = 150(1 - \frac{t}{5})^2$$

$$u = 1 - \frac{t}{5} = 1 - \frac{t}{5} +$$

$$\frac{du}{dt} = -\frac{1}{5} < -60u$$

$$V(u) = 150u^2 > -60(1 - \frac{t}{5})$$

$$V'(u) = 300u$$
\[ R(291) - R(290) \]
\[ \frac{650}{4} (291) - \frac{291^2}{4} - \frac{650}{4} (290) + \frac{290^2}{4} \]

\[ = 26117.25 - 26100 \]
\[ = 17.25 \]
R = (250 + 20n) (100 - 5n)